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International trade, technical change and wage inequality in the U.K. economy

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Contents

Abstract

This paper examines the joint impact of international trade and technical change on U.K. wages across different skill groups. International trade is measured as changes in product prices and technical change as total factor productivity (TFP) growth. We take account of a multi-sector and multi-factor of production economy and use mandated wage methodology to offer a close theoretical-empirical relationship. We use data of the EU KLEMS database and analyse the impact of both, product price changes and TFP changes of 11 U.K. manufacturing sectors on factor rewards of high-, medium- and low-skilled workers. Results show that real wages of skill groups are driven by the sector bias of price change and TFP growth of selected sectors of production. Furthermore, for each year 1970-2005 we estimate the share of the three different skill groups on added value which indicate structural change in the U.K. economy. Empirical results show a structural change in the U.K. economy by the declined share of low-skilled workers and the increased share of medium-skilled and high-skilled workers over the years.

Zusammenfassung

In diesem Beitrag wird der simultane Einfluss internationalen Handels und technologischen Fortschritts auf die Löhne verschieden qualifizierter Arbeitnehmergruppen analysiert. Dabei wird internationaler Handel anhand Produktpreisänderungen und technologischer Fortschritt anhand des Wachstums der totalen Faktorproduktivität in Industrien gemessen. Das theoretische Modell berücksichtigt eine beliebige Anzahl an Produktionsfaktoren und Sektoren. Die Lohngleichung des Modells weist Eigenschaften auf, welche einen engen Bezug zwischen empirischer Spezifikation und Theorie ermöglichen. Mit statistischem Material der EU KLEMS Datenbank wird der Einfluss aus Produktpreisänderungen und Wachstum der totalen Faktorproduktivität von 11 Industrien des verarbeitenden Gewerbes auf die Reallöhne hoch, mittel und gering qualifizierter Arbeitnehmer in Großbritannien untersucht. Die Ergebnisse zeigen, welche Industrien einen wesentlichen Einfluss auf die Reallöhne aufweisen. Desweiteren wird für jedes Jahr 1970-2005 der Anteil der verschieden qualifizierten Arbeitnehmergruppen an der Wertschöpfung der Ökonomie berechnet. Dadurch wird der Strukturwandel in diesem Zeitraum in Großbritannien aufgezeigt. Hierbei wird die rückläufige Nachfrage nach gering qualifizierten Arbeitnehmern und die gestiegene Nachfrage nach mittel und hoch qualifizierten Arbeitnehmern durch die empirischen Ergebnisse deutlich.

JEL classification: F11, F16, J31

Keywords: international trade, technological change, wage differentials

1 Introduction

During the short time period of the last three decades U.K. high-skilled wages have risen very sharply relative to low-skilled wages (see Figure 1). A large variety of literature documents this substantial widening of the U.K. wage structure since the late 1970s (see Desjonqueres et al., 1999; Machin/Reenen, 1998; Schmitt, 1996; Gosling et al., 2000). These structural changes translated into a rise in both, household income inequality and consumption inequality and hence into an increase in the disparities of economic well-being for the families (see Karoly/Burtless, 1995).

The two leading demand-side causes enhancing wage inequality that have been mainly explored are international trade and technical change. According to the first, analogue to the Stolper-Samuelson theorem (Stolper/Samuelson, 1941), a product price change due to export opportunities for a country has strong distributional consequences, making some people worse off and some better off even though there are gains from trade in overall sense. According to the second, the general prospect is that technical change raises the demand for workers with high qualification relative to workers with low qualification (see e.g. Nelson/Phelps, 1966; Griliches, 1969). Hereby, studies argue that the surge of inequality since the late 1970s in U.K. economy, reflected by the ongoing rise in the demand for skilled workers, intensified during the 1980s with the onset of the computer revolution.

Figure 1: U.K. Wage Inequality across Qualification

Source: EU KLEMS Database 2008, own calculation.

A number of studies analyse the effects of international trade and technical change on wage inequality across skill classes. As for international trade, e.g. Murphy/Welch (1991), as a work of labour economists, and Shatz/Sachs (1994, 1998), relying on factor content analysis, document international trade to be the driving force behind wage inequality.[1] Es-

[1] Further examples of factor content methodology are given by Katz/Murphy (1992), Bound/Johnson (1992),

pecially for the U.K. economy, Desjonqueres, et al. (1999) and Neven/Wyplosz (1996) find, contrary to the intuition of Stolper-Samuelson theorem, no evidence for falling relative prices for low-skilled intensive products during the 1980s. In contrast, Haskel/Slaughter (2001) find changes in prices to be the major force behind the rise in U.K. wage inequality in the 1980s. The authors rely on mandated wage methodology. Thereby, the advantage in contrast to earlier studies exists in the close relationship between empirical analysis and underlying theory.[2]

As for technical change, Lawrence et al. (1993), as a work by trade economists, find strong inconsistency with the predictions of Stolper-Samuelson theorem and argue that the observed patterns appear consistent with skill biased technical change across sectors.[3] Using mandated wage methodology, Baldwin/Cain (2000) find only relatively small trade mandated effects on the skill premium, measured by the rise in the relative wage of skilled to less skilled workers, and conclude that technological change appears to have a more likely cause of the rising skill premium than trade. Especially for the U.K. economy, Haskel/Slaughter (2001) find that industry concentration significantly raised skill premia through its effect on total factor productivity (TFP) growth in the 1980s.

As far as we know, none of the studies on neoclassical general equilibrium trade models formalise the joint impact of international trade and technical change on wages. But as both, international trade and technological change seem to be major forces behind wage inequality trends, it is worth knowing the impact of the joint force of trade and technological change on wages. To take this into consideration, we formalise a neoclassical general equilibrium model taking account of the simultaneous impact of the two forces on wages of different skill groups. Thereby, we apply mandated wage methodology. This ensures a strong link between empirical analysis and the framework of Heckscher-Ohlin theorem. International trade is measured as changes in product prices and technical change as TFP growth. Further, previous studies on mandated wage analysis show inconsistency between resulting equations within theoretical analysis and underlying equations of empirical analysis. Hence, a further advantage of this paper is that we aim to perform empirical analysis based on the resulting equations of the neoclassical general equilibrium trade model.

Regarding theoretical analysis, we trace back to Haskel/Slaughter (2001) in accounting for technical changes across sectors in a multi-sector economy. By this means, technological changes influence wages through cross-sector shifts in relative profitability analogue to the effects of changing product prices across sectors. According Stolper Samuelson theorem changing product prices across sectors affect profits across sectors and hence induce wage adjustments. That means, in contrast to the literature on one-sector economies in which factor-biased technical change affects relative wages, Haskel/Slaughter (2001) rely on a multi-sector model in which wage effects of technical change depend on the sector bias, i.e. on what sectors enjoy the most technological progress. Further, analogue to Haskel/Slaughter (2001), we focus on TFP growth since it captures all types of technical change: skill biased technical change as well as factor-neutral and other factor-biased tech-

Johnson/Stafford (1993) and Berman/Bound/Machin (1998).

[2] Further references concerning mandated wage methodology can be found in Slaughter (2000).

[3] For similar results see Berman/Bound/Griliches (1994), Bhagwati/Dehejia (1994) and Lawrence (1996).

nology changes.[4] Based on this framework, we account for the joint force of international trade and technological change on wages of different skill groups. We use mandated wage methodology and provide in a first step a model of an economy of two sectors and two factors of production. Continuative, we focus on an economy of an arbitrary number of sectors and factors of production. The factors of production differ in their skills. The sectors of production differ in their skill intensity. We assume exogenous prices and technology changes. Inter-sectoral factor mobility ensures zero-profits.[5]

Regarding empirical analysis, we rely on the results of the theoretical analysis of this paper and control for the simultaneous impact of international trade and technological change on wages of different skill groups. We analyse the U.K. economy since this economy is characterised by low trade unions density and the lack of collective agreements within the private sector (Grainger/Crowther, 2007). We focus on 11 manufacturing sectors and distinguish between the three skill groups of low-skilled, medium-skilled and high-skilled workers. Using EU KLEMS database we study the impact of value added price change and TFP growth of the 11 manufacturing sectors on average real wages of the different skill groups. Along Stolper-Samuelson theorem, we interpret the results as larger price increases in a sector tend to raise (reduce) the relative wages of factors employed relatively intensively (unintensively) in that sector. For the case of a multi-sector economy, Jones/Scheinkman (1977) state that there must be a good such that an increase in the price of that good will lower the real return to the factor and Ethier (1984) states that for each factor there need not exist a good such that increasing the price of that good will raise the real return to the factor. Findlay/Grubert (1959) bring out the concept of the sector bias of technical change and the effects on real returns to factors. Furthermore, we estimate for each year in consideration (1970 until 2005) the relative shares in total product of each of the three different skill groups. The changes of these shares on added value over time indicate the structural change in the U.K. economy.

To sum up the advantages of this paper: First, we examine the joint impact of international trade and technical change on wages of different skill groups. Second, we not only investigate wage inequality of high-skilled to low-skilled workers but also take into consideration medium-skilled workers. Third, there is consistency between derived equations within theoretical analysis and underlying equations of empirical analysis. Based on mandated wage methodology empirical analysis offers a close relationship to Heckscher-Ohlin framework. Fourth, the empirical analysis aims a) to estimate the impact of sector's product prices and technology on factor returns and b) to estimate the relative shares of factors of production in total product of sectors which indicates structural changes in the economy. Finally, we enhance literature on U.K. economy since we test for empirical relevance of the model using data of three different skill groups for 11 U.K. manufacturing sectors.

The paper is structured as follows. Section 2 comprises the theoretical model based on mandated wage methodology. Thereby 2.1 illustrates the special case of an economy of

[4] For further references see Berndt/Wood (1982) and Morrison Paul (1999).

[5] The framework considers only the production side of only one single economy. Thus the model does not imply factor price equalisation along Samuelson (1948). The analysis imposes no assumptions on cross-country similarities or on consumption side.

two sectors and two factors of production. Continuative in 2.2 it follows the analysis of the general case of an arbitrary number of sectors and factors of production. This gives a theoretical answer to the question of how wages are affected by international trade and technical change. Section 3 comprises the empirical analysis, whereby 3.1 illustrates the econometric specification, 3.2 describes the used statistical data and 3.3 the empirical results. Section 4 concludes.

2 Mandated Wage Methodology

2.1 The Special Case of Two Sectors and Two Factors of Production

To illustrate the coherence between international trade, technical change and wages we first restrict the attention to the special case of an economy of two sectors and two factors of production, namely high-skilled and low-skilled workers, before generalisation of the model to M sectors and N factors of production. One of the sectors is assumed to be high-skill intensitve. In an economy in which sectors differ in their skill intensity, the demand for high-skilled workers relative to low-skilled workers changes whenever workers move across sectors. E.g. if workers move to the high-skill intensive sector then their demand relative to the demand for low-skilled workers rises. The workers migration flows are endogenously caused by inter-sectoral profitability. These changes in profitability are in turn caused by changes in product prices or technology. To formalise this process we derive the wage equations of an economy in what follows.

The economy under consideration is characterised by perfect competition in the product markets and factor markets. The two internationally immobile factors of production, V_j, produce two tradable goods, Y_i, of sector i $(i = 1, 2)$. The factors of production V_j comprise workers of different qualification j $(j = 1, 2)$. Capital is supposed to be internationally mobile and therefore remains disregarded in the further discussion on determinants of wages. Moreover, the model does not consider non-traded goods as wages of these sectors are determined by the zero-profit conditions of tradable sectors. We assume full mobility of workers between the sectors. This assures a long run point of view. Fully flexible factor prices ensure full employment of the exogenously given respective endowments. Restricting the attention to a small single economy, product prices, p_i $(i = 1, 2)$, are given exogenously. Besides, we assume exogenous technology changes. We assume the empirically more plausible case of sector biased (Hicks-neutral) technological progress which drives the economy (see Haskel/Slaughter, 2002). That means a proportional reduction of input requirement across all factors, within a given sector, but with different degrees of improvement across sectors. The formalisation of TFP growth captures skill biased technical change as well as factor-neutral or other factor-biased technology changes. With the small-economy assumption TFP growth does not affect prices.

For each sector the firm's production functions are characterised by the properties of Cobb-Douglas production functions and hence are given by

$$\begin{aligned} Y_1 &= A_1 V_{11}^{\alpha_{11}} V_{12}^{\alpha_{12}} \\ Y_2 &= A_2 V_{21}^{\alpha_{21}} V_{22}^{\alpha_{22}}. \end{aligned} \tag{1}$$

Thereby, TFP is denoted by the positive constant A. The exponent α of each input variable indicates the relative share of that input in total product. Assuming constant returns to scale it follows

$$\begin{aligned} \alpha_{11} &= 1 - \alpha_{12} \\ \alpha_{22} &= 1 - \alpha_{21}, \end{aligned} \tag{2}$$

whereby $0 < \alpha_{ij} < 1$ and $\alpha_{ij} \neq \alpha_{kj}$ so that each factor of production acts as an input in each sector.

The amount of factors employed in each sector is constrained by the endowments V_1 and V_2 available in the economy. The resource constraints state that

$$\begin{aligned} V_{11} + V_{21} &= V_1 \\ V_{12} + V_{22} &= V_2. \end{aligned} \tag{3}$$

Firms maximise their profits π_i $(i = 1, 2)$ in choosing their optimal inputs given the prices and the ressource constraints. They only face factor prices w_j as the costs of production. They face the following problem of constrained maximisation

$$\begin{aligned} \pi_1 &= p_1 A_1 V_{11}^{\alpha_{11}} V_{12}^{\alpha_{12}} - w_1 V_{11} - w_2 V_{12} \longrightarrow \max_{V_{11}, V_{12}} \\ \pi_2 &= p_2 A_2 V_{21}^{\alpha_{21}} V_{22}^{\alpha_{22}} - w_1 V_{21} - w_2 V_{22} \longrightarrow \max_{V_{21}, V_{22}}, \end{aligned} \tag{4}$$

subject to resource constraints (3). Solving the firms problem of maximisation yields factor prices w_1 and w_2 of factors of production V_1 and V_2 of sector 1

$$\begin{aligned} w_1 &= p_1 Y_1 \alpha_{11} V_{11}^{-1} \\ w_2 &= p_1 Y_1 \alpha_{12} V_{12}^{-1} \end{aligned} \tag{5}$$

and of sector 2

$$\begin{aligned} w_1 &= p_2 Y_2 \alpha_{21} V_{21}^{-1} \\ w_2 &= p_2 Y_2 \alpha_{22} V_{22}^{-1}. \end{aligned} \tag{6}$$

The set of equilibrium conditions for the economy is that profits equal zero and that full employment of all resources is given. Under perfect competition the value of marginal product of a factor equals to that factor's reward which in turn is equal in each sector. Based on this, we deduce the cost functions of sectors of the form that product prices p_i

$(i = 1, 2)$ are functions of TFP A_i $(i = 1, 2)$ and functions of factor prices w_j $(j = 1, 2)$. Appendix A.1 details the transformation to the cost functions by applying factor price ratios and factor intensities. The cost functions of sectors 1 and 2 reveal as

$$\begin{aligned} p_1 &= \frac{1}{A_1} \frac{w_2^{\alpha_{12}} w_1^{(1-\alpha_{12})}}{\alpha_{12}^{\alpha_{12}} (1-\alpha_{12})^{(1-\alpha_{12})}} \\ p_2 &= \frac{1}{A_2} \frac{w_1^{\alpha_{21}} w_2^{(1-\alpha_{21})}}{\alpha_{21}^{\alpha_{21}} (1-\alpha_{21})^{(1-\alpha_{21})}}. \end{aligned} \tag{7}$$

Getting additive coherence, we use logarithmic calculus of functions (7). Further, taking account of (2) and denoting $\overline{p_i} = A_i p_i \alpha_{i1}^{\alpha_{i1}} \alpha_{i2}^{\alpha_{i2}}$ and hence, $\ln \overline{p_i} = \ln(A_i p_i) + \alpha_{i1} \ln \alpha_{i1} + \alpha_{i2} \ln \alpha_{i2}$ $(i = 1, 2)$ we obtain the equation system

$$\begin{aligned} \ln \overline{p_1} &= \alpha_{11} \ln w_1 + \alpha_{12} \ln w_2 \\ \ln \overline{p_2} &= \alpha_{21} \ln w_1 + \alpha_{22} \ln w_2. \end{aligned} \tag{8}$$

Now, to derive the factor prices w_j of the economy needs solving system (8). Solving this system by Cramer's Rule yields

$$\begin{aligned} \ln w_1 &= \gamma_1 + \sum_{i=1}^{2} \beta_{i1} \ln A_i + \sum_{i=1}^{2} \beta_{i1} \ln p_i \\ \ln w_2 &= \gamma_2 + \sum_{i=1}^{2} \beta_{i2} \ln A_i + \sum_{i=1}^{2} \beta_{i2} \ln p_i, \end{aligned} \tag{9}$$

whereby, $\beta_{ij} = \Phi_{ij}/\Phi$ and $\gamma_j = \frac{1}{\Phi} \prod_{1=1}^{2} \left[\prod_{k=1}^{2} \alpha_{ik}^{\alpha_{ik}} \right]^{\Phi_{ij}}$. Appendix A.2 details the determination of factor prices of a $2 * 2$ economy. Equation system (9) determines the factor prices w_1 and w_2 of the economy. Because of factor mobility across sectors, factor prices are not indexed by sector i. The factor prices are functions of TFP of the two sectors, A_1 and A_2, and are functions of exogenous given product prices p_1 and p_2. To answer the question how product price and technology changes affect wages, we differentiate equations (9) totally. This yields

$$\begin{aligned} d \ln w_1 &= \frac{dw_1}{w_1} = \sum_{i=1}^{2} \beta_{i1} \frac{dA_i}{A_i} + \sum_{i=1}^{2} \beta_{i1} \frac{dp_i}{p_i} \\ d \ln w_2 &= \frac{dw_2}{w_2} = \sum_{i=1}^{2} \beta_{i2} \frac{dA_i}{A_i} + \sum_{i=1}^{2} \beta_{i2} \frac{dp_i}{p_i}. \end{aligned} \tag{10}$$

Denoting the percentage changes by $dw_j/w_j = \hat{w}_j$ $(j = 1, 2)$, $dA_i/A_i = \hat{A}_i$ and $dp_i/p_i = \hat{p}_i$ $(i = 1, 2)$ it follows

$$\begin{aligned} \hat{w}_1 &= \sum_{i=1}^{2} \beta_{i1} \hat{A}_i + \sum_{i=1}^{2} \beta_{i1} \hat{p}_i \\ \hat{w}_2 &= \sum_{i=1}^{2} \beta_{i2} \hat{A}_i + \sum_{i=1}^{2} \beta_{i2} \hat{p}_i. \end{aligned} \tag{11}$$

Equations (11) indicate that subsequent to a change in TFP of the two sectors, A_1 and A_2, or product prices, p_1 and p_2, competition will serve to constrain a change in wages w_1 and w_2. So the equations show how factor prices adjust to changes in product prices or technical change to restore zero profits in the sectors. Being the two factors of production high-skilled and low-skilled workers, the equations capture the wage adjustment to shifts in the demand for high-skilled workers relative to low-skilled workers. The wage effects depend on the sector bias of product price and technical changes. Stolper-Samuelson theorem predicts that product price increases in a sector will raise (reduce) the relative wages of factors employed relatively intensively (unintensively) in that sector. Similarly, technical change is expected to influence wages through shifts in relative profitability (see Findlay/Grubert, 1959).

2.2 The General Case of M Sectors and N Factors of Production

In what follows we presume an economy of an arbitrary number of sectors and factors of production and discuss the coherence between international trade, technical change and wages. That means, there are an arbitrary number of workers of different qualification. The arbitrary number of sectors differ in their skill intensity. In the same way as mentioned in the previous section, in a multi-sector economy, product prices changes across sectors affect firm's profits across sectors and hence induce wage changes to restore economy-wide competitive equilibrium. Analogue, in a multi-sector model, technological progress is expected to influence wages through cross-sector shifts in relative profitability (see Haskel/Slaughter, 2001).

As before, the economy is characterised by perfect competition in the product and factor markets. Now, there are N internationally immobile factors of production V_j $(j = 1, .., n)$ and M tradable goods Y_i $(i = 1, .., m)$. Again, capital is supposed to be internationally mobile and therefore remains disregarded in the further discussion. Further, we do not consider non-traded goods as wages of these sectors are determined by the conditions of tradable sectors. Equally to the previous section, factors of production are assumed to be mobile between the sectors and fully flexible factor prices ensure full employment of the exogenously given respective endowments. Furthermore, product prices p_i $(i = 1, .., m)$ are given exogenously, hence they are also the world prices. The sector biased (Hicks-neutral) technological progress is assumed to be exogenous and is measured as TFP growth.

For each sector of production Y_i with $\Sigma_{j=1}^{n}\alpha_{ij} = 1$ and $0 < \alpha_{ij} < 1$ $(\forall\ i = 1, ..., m)$ the Cobb-Douglas production function is given by

$$Y_i = A_i \Pi_{j=1}^{n} V_{ij}^{\alpha_{ij}}. \tag{12}$$

The resource constraints state as

$$V_j = \Sigma_{i=1}^{n} V_{ij}, \tag{13}$$

where V_j $(j = 1, ..., n)$ is the total amount of each factor of production in the economy.

Firms maximise their profits π_i $(\forall\ i = 1, ..., m)$ and only face factor prices w_j $(\forall\ j = 1, ..., n)$ as their production costs. They face the following problem of constrained maximisation

$$\pi_i = p_i A_i \Pi_{j=1}^{n} V_{ij}^{\alpha_{ij}} - \Sigma_{j=1}^{n} w_j V_{ij} \longrightarrow \max_{V_{ij}} \tag{14}$$

subject to resource constraints (13). Solving the firms problem of maximisation yields the factor price w_k for factor k $(\forall\ k = 1, ..., N)$

$$w_k = p_i Y_i \alpha_{ik} V_{ik}^{-1}. \tag{15}$$

Analogue to the previous section of two goods and two factors, in equilibrium profits equal zero and all resources are fully employed. The value of marginal product of a factor equals to that factor's reward which is equal in each sector. The cost functions reveal as

$$p_i = \frac{1}{A_i} \Pi_{j=1}^{n} \frac{w_j^{\alpha_{ij}}}{\alpha_{ij}^{\alpha_{ij}}}. \tag{16}$$

whereby product prices p_i $(\forall\ i = 1, ..., m)$ are functions of TFP A_i $(\forall\ i = 1, ..., m)$ and of factor prices w_j $(\forall\ j = 1, ..., n)$. Appendix A.3 details the transformation using factor price ratios and factor intensities.

Using logarithmic calculus of (16) to get additive coherence and denoting $\overline{p_i} = A_i p_i \Pi_{j=1}^{n} \alpha_{ij}^{\alpha_{ij}}$ we obtain the equation system

$$\ln \overline{p_i} = \Sigma_{j=1}^{n} \alpha_{ij} \ln w_j. \tag{17}$$

Solving system (17) by Cramer's Rule yields factor prices w_j $(\forall\ j = 1, ..., n)$ of the economy. A unique solution requires the assumption that the number of sectors of production M equals the number of factors of production N. Denoting $\Phi = \det(\underline{\Phi})$, $\beta_{ij} = \Phi_{ij}/\Phi$, $\alpha_j = \Pi_{i=1}^{n} (\Pi_{k=1}^{n} \alpha_{ik}^{\alpha_{ik}})^{\Phi_{ij}}$, $\gamma_j = \frac{1}{\Phi} \ln \alpha_j$ and $\underline{\Phi}$ is the matrix of $\alpha-$ components, the factor prices of the economy reveal as

$$\ln w_j = \gamma_j + \Sigma_{i=1}^{n} \beta_{ij} \ln A_i + \Sigma_{i=1}^{n} \beta_{ij} \ln p_i. \tag{18}$$

Appendix A.4 details the determination of factor prices of a $M * N$ economy. With equation system (18) all factor prices w_j of the economy are determined. The crucial step consists of the derivation of factor prices w_j depending on TFP A_i and on exogenous product prices p_i of the whole economy. Totally differentiating (18) gives

$$\frac{dw_j}{w_j} = \Sigma_{i=1}^{n} \beta_{ij} \frac{dA_i}{A_i} + \Sigma_{i=1}^{n} \beta_{ij} \frac{dp_i}{p_i}. \tag{19}$$

Denoting the percentage changes by $dw_j/w_j = \hat{w}_j$ $(\forall\ j = 1, ..., n)$, $dA_i/A_i = \hat{A}_i$ and $dp_i/p_i = \hat{p}_i$ $(\forall\ i = 1, ..., m)$ it follows

$$\hat{w}_j = \Sigma_{i=1}^{n} \beta_{ij} \left(\hat{A}_i + \hat{p}_i \right), \tag{20}$$

where $\hat{A}_i$ denotes the TFP growth and $\hat{p}_i$ the change in the product price of sectors. The change in the wage of factor j is denoted by $\hat{w}_j$, which again is identical economy-wide since all factors are mobile across sectors. Equation (20) indicates that factor prices adjust to changes in product prices or technology to restore zero-profits in all sectors of the economy. Analogue to the previous section, the wage effects depend on the sector bias of product price and technical changes. By the essence of Stolper-Samuelson theorem and by the intuition of Findlay/Grubert (1959), larger price and technology increases in a sector tend to raise (reduce) the relative wages of factors employed relatively intensively (unintensively) in that sector. Furthermore, Jones/Scheinkman (1977) convey that in the case of a multi-sector and multi-factor economy there must be a good such that an increase in the price of that good will lower the real return to the factor. And according Ethier (1984), for each factor there need not exist a good such that increasing the price of that good will raise the real return to the factor. If the number of sectors M does not equal the number of factors N the derivation of distributional consequences is much more complicated. In the uneven case of more factors than sectors the theorem according Jones/Scheinkman (1977) no longer holds. The special case of two goods and three factors, in the sense of one immobile factor, has been analysed by Jones (1971), Mayer (1974), Mussa (1974) and Neary (1978). The change in real return of the mobile factor depends on the relative consumption of the goods. The case of more sectors than factors results in multiple solutions for the output of sectors. The generalization of this case has been analysed by Dornbush/Fischer/Samuelson (1980).

3 Empirical Analysis

3.1 Econometric Specification

To answer the question how wage changes of different skill groups depend on international trade and technological progress, we analysed the determination of factor prices of a multi-sector economy in previous sections. Based on this theoretical background we can now derive the econometric specifications. First, we estimate the impact of international trade, measured as changes in product prices, and technical change, measured as TFP growth, on factor returns. Second, we derive the equation specification to estimate relative shares of factors in total product of sectors. These relative shares over time indicate the structural change in the economy.

To estimate the impact of international trade and technical change on factor returns we rely on (20) and regress

$$\hat{w}_j = \beta_{1j}\hat{A}_1 + \beta_{2j}\hat{A}_2 + .. + \beta_{mj}\hat{A}_m + \beta_{1j}\hat{p}_1 + \beta_{2j}\hat{p}_2 + .. + \beta_{mj}\hat{p}_m + \varepsilon_j, \qquad (21)$$

where ε_j is an additive error term. We treat $\hat{A}_i$ and $\hat{p}_i$ $(i = 1, .., m)$ exogenous since we consider a small open economy and exogenous technical change, and estimate their sector bias. With (21) we estimate the impact of each sectors product price and technical changes on economy-wide factor prices. The coefficients β_{ij} are the parameters to be estimated.

To estimate the relative shares of factors in total product we proceed as follows. According Cobb-Douglas production functions (1) and (12) the relative shares of inputs in total product are denoted by α_{ij}. Within the equation systems (9) and (18) these shares are combined in parameters β_{ij}. Thereby, the parameters β_{ij} are functions of parameters α_{ij}. In the following we revert from parameters β_{ij} of the reduced forms (9) and (18) to parameter α_{ij} of the structural forms (8) and (17).

Regarding the model of a two sector and two factor economy, we revert from parameters β_{ij} to parameters α_{ij} by using (9) and subtract the factor price 1 by factor price 2. This yields

$$\ln w_1 - \ln w_2 = \frac{1}{1-\alpha_{12}-\alpha_{21}}(\ln A_1 p_1 - \ln A_2 p_2) + \left[\frac{\alpha_{12}\ln\alpha_{12} - \alpha_{21}\ln\alpha_{21} + (1-\alpha_{12})\ln(1-\alpha_{12}) + (\alpha_{12}-1)\ln(1-\alpha_{21})}{1-\alpha_{12}-\alpha_{21}}\right]. \quad (22)$$

Denoting $D = 1 - \alpha_{12} - \alpha_{21}$ it follows

$$\ln w_1 - \ln w_2 = \frac{1}{D}\cdot(\ln A_1 p_1 - \ln A_2 p_2) + [...]. \quad (23)$$

Solving this linear regression model (23) gives D and hence the parameters α_{ij} $(i = 1, 2$ and $j = 1, 2)$

$$\alpha_{12} = (1 - \alpha_{21} - D). \quad (24)$$

Inserting (24) into the second term of the right hand side of equation (22) and denoting E gives

$$E = \frac{1}{D}(\alpha_{12}\ln\alpha_{12} - \alpha_{21}\ln\alpha_{21} + (1-\alpha_{12})\ln(1-\alpha_{12}) + (\alpha_{12}-1)\ln(1-\alpha_{21})) \quad (25)$$

and hence it follows

$$\begin{aligned} D\cdot E &= (1-\alpha_{21}-D)\ln(1-\alpha_{21}-D) - \alpha_{21}\ln\alpha_{21} \\ &+ (\alpha_{21}+D)\ln(\alpha_{21}+D) + (-\alpha_{21}-D)\ln(1-\alpha_{21}), \end{aligned} \quad (26)$$

whereby finally parameters α_{12} and α_{21} are calculated. Further, using condition (2) gives α_{ij} $(i = 1, 2; j = 1, 2)$ of the $2 * 2$ economy.

Regarding the model of M sector and N factor economy, we revert from parameters β_{ij} to parameters α_{ij} by using (17). The left hand side of (17) is given by

$$\begin{aligned} \ln(A_i p_i \alpha_{i1}^{\alpha_{i1}} ... \alpha_{in}^{\alpha_{in}}) &= \underbrace{\ln(A_i p_i)} + \underbrace{\alpha_{i1}\ln\alpha_{i1} + .. + \alpha_{in}\ln\alpha_{in}} \\ &= \quad y_i \quad - \quad H(\alpha_{i1}, .., \alpha_{in}) \end{aligned} \quad (27)$$

with $y_i = \ln(A_i p_i)$ observations and entropy of discrete probability distribution $(v_1, .., v_n)$. With $0 \leq H(v_1, .., v_n) \leq \ln n$ the maximum value reveals as $\ln n$, if $v_1 = .. = v_n = \frac{1}{n}$. Hence it follows

$$H(\alpha_{i1}, .., \alpha_{in}) = -v_1 \ln v_1 - .. - v_n \ln v_n. \quad (28)$$

Denoting $x_i = \ln w_i$ equation k of system (17) reveals as

$$\begin{aligned} \alpha_{k1}x_1 + .. + \alpha_{kn}x_n &= y_k - H(\alpha_{k1}, .., \alpha_{kn}) \\ \underbrace{\Sigma_{i=1}^n \alpha_{ki}x_i + H(\alpha_{k1}, .., \alpha_{kn})} &= y_k. \\ f(x_1, .., x_n; \alpha_{k1}, .., \alpha_{kn}) &= y_k \end{aligned} \quad (29)$$

This is now the system we will estimate to calculate the relative shares of each factor in total product of the economy. With equation (29) y_k is only dependent on parameter α of row k. That means, the parameters of equation k are estimated by solving the nonlinear multiple regression model

$$f(x_1, .., x_n; \alpha_{k1}, .., \alpha_{kn}) = y_k. \quad (30)$$

The problem of optimisation yields as

$$S(\alpha_{k1}, .., \alpha_{kn}) = \Sigma_{j=1}^N (y_k^j - f(x_1^j, .., x_n^j; \alpha_{k1}, .., \alpha_{kn}))^2 \quad (31)$$

subject to

$$\begin{aligned} \alpha_{kn} &= 1 - (\alpha_{k1} + .. + \alpha_{k,n-1}) \geq 0 \\ \alpha_{ki} &\geq 0. \end{aligned} \quad (32)$$

This nonlinear multiple regression model is transformed in a linear regression model if $H(\alpha_{k1}, .., \alpha_{kn})$ of equation (29) is regarded as the constant term in the regression model. We estimate the parameters α_{ki} applying a regression with interval constraints so that $0 < \alpha_{ki} < 1$. Hence, we derived the equation specification to estimate the relative shares of factors in total product of sectors. These shares considered over time indicate the structural change in the economy.

3.2 Data

The main source is the March 2008 release of EU KLEMS database. It provides data for the years 1970 up to 2005 and differentiated by skill groups. We use data on hourly labour compensation of high-skilled, medium-skilled and low-skilled workers. Hereby, educational attainment is categorised as follows: university graduates, intermediate and workers with no formal qualifications. Gross output price indices are used to transform nominal values in real values. The dependent variables of the regression analysis are the real wages of high-skilled, medium-skilled and low-skilled workers, calculated as average real wages over time of the economy. Furthermore, we use data on hours worked of the different skill groups to estimate an equation with interval constraints.

Moreover, the database provides gross value added price indices. As value added is the residual of gross output minus intermediate inputs, value added prices display the difference between gross output prices and intermediate prices. We use data on value added based TFP growth. TFP growth is measured as Solow residual, known as that part of output growth that cannot be accounted for by the growth of primary factors of production.

Regarding value added prices and TFP growth we use the provided data on 11 manufacturing sectors. These are the independent variables of the regression analysis, provided for the years 1970-2005.

Table 1 reports summary statistics for the variables under consideration. It shows the growth rates and average annual growth rates for the years 1970 to 2005. Real wages of medium-skilled workers exhibit the highest growth rate as well as the highest average annual growth rate from 1970 to 2005 among the different skill groups. Nevertheless, real wages of high-skilled and low-skilled workers are also characterised by positive growth rates underlying the wage inequality in Figure 1. Value added based TFP growth is characterised by positive growth rates while value added prices declined over 1970 to 2005 in most manufacturing sectors.

Table 1: Summary Statistics

	Average Annual Growth Rate 1970-2005 (in %)	Growth Rate 1970-2005 (in %)
Real Wages of High-Skilled Workers	0.97	40.25
Real Wages of Medium-Skilled Workers	1.79	86.29
Real Wages of Low-Skilled Workers	0.92	37.91
Value Added Prices of Sector		
Food, Beverages and Tobacco	0.35	12.92
Textiles, Leather and Footwear	-0.04	-1.51
Wood and Cork	0.69	27.20
Pulp, Paper, Printing and Publishing	0.30	10.86
Chemical, Rubber, Plastics and Fuel	-1.07	-31.41
Other Non-Metallic Mineral	-0.26	-8.80
Basic Metals and Fabricated Metal	-0.52	-16.73
Machinery, nec	-0.16	-5.36
Electrical and Optical Equipment	-0.95	-28.49
Transport Equipment	-0.19	-6.56
Manufacturing nec; Recycling	1.80	86.77
Value Added Based TFP Growth of Sector		
Food, Beverages and Tobacco	0.44	16.44
Textiles, Leather and Footwear	1.96	97.30
Wood and Cork	0.43	16.07
Pulp, Paper, Printing and Publishing	0.72	28.72
Chemical, Rubber, Plastics and Fuel	2.56	142.58
Other Non-Metallic Mineral	1.83	88.75
Basic Metals and Fabricated Metal	1.90	93.11
Machinery, nec	0.91	37.29
Electrical and Optical Equipment	3.46	228.43
Transport Equipment	2.17	112.33
Manufacturing nec; Recycling	-2.01	-50.82

Source: EU KLEMS Database 2008, own calculation.

3.3 Empirical Results

Table 2 reports estimation results for equation (21). It shows coefficient estimates for the regressors $\hat{A}_i$ and $\hat{p}_i$. These coefficients indicate to what extent the sector bias of value added price and value added based TFP growth mandate real wage changes to maintain zero profits in all sectors.

A 1 % change of value added price of Textiles, Leather and Footwear mandated an insignificant 0.77 % rise in high-skilled real wages. As to high-skilled workers it is the sector where sector bias of price change had the largest positive impact on real wages. Regarding the real wages of medium-skilled workers it is the sector of Pulp, Paper, Printing and Publishing where price changes result in extensive positive real wage changes. A 1 % change of value added price of Pulp, Paper, Printing and Publishing mandated a significant 1.1 % rise in medium-skilled real wages. As well it is the sector with the largest positive impact of price changes on low-skilled real wages. Here a 1 % change of value added price mandated an insignificant 0.92 % rise in low-skilled real wages.

Comparison among the skill groups shows that there are sectors where value added price changes result in positive or negative real wage changes for all three different skill groups (Sector of Textiles, Leather and Footwear; Pulp, Paper, Printing and Publishing; Chemical, Rubber, Plastics and Fuel; Other Non-Metallic Mineral and the sector of Manufacturing nec; Recycling). Against it, results show that there are sectors where value added price changes result in positive and negative real wage changes among the different skill groups (Sector of Food, Beverages and Tobacco; Wood and Cork; Basic Metals and Fabricated Metal; Machinery, nec; Electrical and Optical Equipment and the sector of Transport Equipment).

Regarding the impact of value added based TFP growth on real wages of different skill groups results show that sector bias in Basic Metals and Fabricated Metal sector had the largest positive impact on high-skilled real wages. Besides, it is the value added based TFP growth of the sector of Other Non-Metallic Mineral which had the largest positive impact on real wages of medium-skilled and low-skilled workers. A 1 % change of value added based TFP growth of Basic Metals and Fabricated Metal sector mandated a significant 0.6 % rise in high-skilled real wages. And further, a 1 % change of value added based TFP growth of Other Non-Metallic Mineral sector mandated a significant 0.3 % rise in medium-skilled real wages and a significant 0.37 % rise in low-skilled real wages.

Comparison among the skill groups shows that there are sectors where value added based TFP growth result in positive or negative real wage changes for all considered skill groups (Sector of Textiles, Leather and Footwear; Chemical, Rubber, Plastics and Fuel; Other Non-Metallic Mineral; Basic Metals and Fabricated Metal and the sector of Machinery, nec) Contrary, Table 2 shows that there are sectors where value added based TFP growth results in positive and negative real wage changes among the different skill groups (Sector of Food, Beverages and Tobacco; Wood and Cork; Pulp, Paper, Printing and Publishing; Electrical and Optical Equipment; Transport Equipment and the sector of Manufacturing nec; Recycling).

Moreover, Table 2 reports the results of Durbin-Watson d-statistic. According Savin/White (1977) the results show that there is no positive or negative first-order autocorrelation. As the used statistical data of variables under consideration are differentiated data according equation (21), we used stationary time series.

Turning to the structural change in the economy Table 3 reports estimated relative shares in total product of sectors of high-skilled, medium-skilled and low-skilled workers over time. These shares on added value are estimated by equation (29).

Table 2: Mandated Wage Regressions

	$\hat{w}_{high-skilled}$	$\hat{w}_{medium-skilled}$	$\hat{w}_{low-skilled}$
Value Added Prices $(\hat{p}_i)$ of Sector			
Food, Beverages and Tobacco	-0.370	-0.262	0.037
	(0.446)	(0.385)	(0.453)
Textiles, Leather and Footwear	0.765	0.403	0.175
	(0.551)	(0.476)	(0.559)
Wood and Cork	-0.272	0.03	-0.242
	(0.350)	(0.301)	(0.354)
Pulp, Paper, Printing and Publishing	0.658	1.062*	0.917
	(0.706)	(0.610)	(0.717)
Chemical, Rubber, Plastics and Fuel	0.198	0.038	0.036
	(0.150)	(0.130)	(0.153)
Other Non-Metallic Mineral	0.003	0.151	0.222
	(0.360)	(0.311)	(0.365)
Basic Metals and Fabricated Metal	0.407	-0.187	-0.123
	(0.365)	(0.316)	(0.371)
Machinery, nec	0.080	-0.038	0.320
	(0.614)	(0.531)	(0.624)
Electrical and Optical Equipment	0.095	-0.201	-0.302
	(0.294)	(0.254)	(0.299)
Transport Equipment	0.538	0.015	-0.426
	(0.358)	(0.309)	(0.363)
Manufacturing nec; Recycling	-0.166	-0.046	-0.173
	(0.164)	(0.141)	(0.166)
Value Added Based TFP Growth $(\hat{A}_i)$ of Sector			
Food, Beverages and Tobacco	0.232	0.207	-0.034
	(0.429)	(0.370)	(0.435)
Textiles, Leather and Footwear	0.355	0.034	0.260
	(0.282)	(0.244)	(0.287)
Wood and Cork	0.075	-0.087	-0.204
	(0.251)	(0.217)	(0.255)
Pulp, Paper, Printing and Publishing	-1.086***	-0.258	0.039
	(0.335)	(0.289)	(0.340)
Chemical, Rubber, Plastics and Fuel	-0.572*	-0.062	-0.229
	(0.308)	(0.266)	(0.313)
Other Non-Metallic Mineral	0.157	0.303*	0.367*
	(0.209)	(0.181)	(0.212)
Basic Metals and Fabricated Metal	0.597**	0.171	0.169
	(0.245)	(0.211)	(0.248)
Machinery, nec	-0.543*	-0.255	-0.240
	(0.263)	(0.227)	(0.267)
Electrical and Optical Equipment	0.512*	0.038	-0.121
	(0.252)	(0.217)	(0.256)
Transport Equipment	0.086	0.153	-1.000
	(0.198)	(0.171)	(0.201)
Manufacturing nec; Recycling	0.119	-0.103	-0.003
	(0.160)	(0.138)	(0.162)
cons	0.765	0.359	0.921
	(1.201)	(1.037)	(1.220)
R-squared	0.7282	0.5789	0.5158
Durbin-Watson d-statistic	2.356	2.073	2.085

Notes: Standard errors in parentheses and p-values $^{*}p < 0.1$, $^{**}p < 0.05$ and $^{***}p < 0.01$.
Source: EU KLEMS Database 2008, own calculation.

As Table 3 shows the share of high-skilled worker rose by 17.80 percentage points in the economy over the years 1970-2005. Against it, the share of medium-skilled worker rose by 27.33 percentage points. Only the group of low-skilled worker is characterised by the huge decline of 45.13 percentage points in this time period. Whilst low-skilled workers accounted with 57.39 % for the largest share among the workforce in 1970, it is the group of medium-

Table 3: Structural Change in the U.K. Economy

Year	High-Skilled Workers		Medium-Skilled Workers		Low-Skilled Workers	
	Relative Shares in Total Product of					
1970	0.11	(0.026)	42.50	(0.026)	57.39	(2.095)
1971	0.51	(0.005)	43.92	(0.439)	55.57	(0.347)
1972	0.90	(0.026)	45.32	(0.026)	53.78	(0.147)
1973	1.28	(0.026)	46.49	(0.026)	52.23	(0.079)
1974	1.72	(0.026)	47.60	(0.026)	50.68	(0.044)
1975	2.17	(0.026)	49.05	(0.026)	48.78	(0.025)
1976	2.59	(0.026)	50.25	(0.026)	47.16	(0.022)
1977	3.17	(0.026)	50.86	(0.026)	45.97	(0.023)
1978	3.77	(0.026)	51.62	(0.026)	44.61	(0.023)
1979	3.85	(0.026)	53.42	(0.026)	42.73	(0.028)
1980	4.16	(0.026)	54.80	(0.026)	41.04	(0.031)
1981	4.52	(0.026)	56.21	(0.026)	39.27	(0.034)
1982	5.08	(0.026)	56.78	(0.026)	38.14	(0.032)
1983	5.67	(0.026)	57.38	(0.026)	36.95	(0.030)
1984	6.13	(0.026)	58.07	(0.026)	35.80	(0.030)
1985	7.03	(0.026)	63.67	(0.026)	29.30	(0.033)
1986	6.39	(0.026)	59.24	(0.026)	31.70	(0.028)
1987	7.17	(0.026)	61.79	(0.026)	31.04	(0.030)
1988	7.52	(0.026)	62.65	(0.026)	29.83	(0.030)
1989	7.69	(0.026)	64.35	(0.026)	27.96	(0.030)
1990	8.27	(0.026)	65.31	(0.026)	26.42	(0.028)
1991	8.50	(0.026)	65.05	(0.026)	26.45	(0.027)
1992	10.24	(0.026)	65.37	(0.026)	24.39	(0.022)
1993	10.84	(0.026)	65.90	(0.026)	23.26	(0.021)
1994	11.05	(0.026)	68.77	(0.026)	20.18	(0.020)
1995	11.78	(0.026)	69.48	(0.026)	18.74	(0.019)
1996	11.98	(0.026)	70.19	(0.026)	17.83	(0.018)
1997	12.49	(0.026)	71.56	(0.026)	15.95	(0.016)
1998	13.18	(0.026)	71.68	(0.026)	15.14	(0.015)
1999	14.15	(0.026)	71.39	(0.026)	14.46	(0.014)
2000	15.05	(0.026)	71.03	(0.026)	13.92	(0.013)
2001	15.54	(0.026)	70.54	(0.026)	13.92	(0.013)
2002	16.10	(0.026)	70.84	(0.026)	13.06	(0.012)
2003	17.03	(0.026)	70.38	(0.026)	12.59	(0.011)
2004	17.72	(0.026)	69.93	(0.026)	12.35	(0.011)
2005	17.91	(0.026)	69.83	(0.026)	12.26	(0.011)

Note: Standard errors in parentheses.
Source: EU KLEMS Database 2008, own calculation.

skilled worker with a share of 69.83 % in 2005. These results show the declined demand for low-skilled workers and the increased demand for high-skilled and medium-skilled workers in the U.K. economy over the years 1970 to 2005.

4 Conclusion

In this paper we examine the joint impact of international trade and technical change on real wages of high-skilled, medium-skilled and low-skilled workers. Thereby, international trade is measured as changes in product prices and technical change as TFP growth. Along Haskel/Slaughter (2001) we focus on the sector bias of price and technical changes in taking account of a multi-sector and multi-factor of production economy. In contrast to previous literature on mandated wage regressions we not only examine the joint impact of trade and technology on wages but provide consistency between theoretical and empirical analysis. First, the empirical analysis aims to estimate the impact of sector's product prices and technology on factor returns and second, it aims to estimate the relative shares of factors of production in total product of sectors which indicates structural changes in the economy.

Results show that real wages of high-skilled, medium-skilled and low-skilled workers are driven by the sector bias of price change and TFP growth of selected sectors of production. The rise of real wage of high-skilled workers over 1970-2005 is mainly affected by the sector bias of Textiles, Leather and Footwear price change and the sector bias of TFP growth of Basic Metals and Fabricated Metal sector. The rise of real wages of medium-skilled and low-skilled workers is mainly affected by the sector bias of Pulp, Paper, Printing and Publishing price change and the sector bias of TFP growth of Other Non-Metallic Mineral sector. To deduce to structural change in the U.K. economy we estimate relative shares of factors in total product of sectors. Here, results show a declined demand for low-skilled workers and a increased demand for medium-skilled and high-skilled workers over the years.

Nevertheless, there are several limitations by using neoclassical models. The factor markets are supposed to be frictionless markets. Hence, we only focus on trade and technology effect on wages by ruling out equilibrium unemployment by assumption. But the political debate about effects of international trade and technical change on labour markets is focused on wage inequality as well as on unemployment. Furthermore, in neoclassical models economic activity takes place in sectors of production. But empirical literature shows that much of the observed reallocation due to increased international trade occurs across firms within industries rather than between industries (see e.g. Levinsohn, 1999; Attanasio/Pinelopi/Pavcnik, 2004). That means, it would be useful for further research on the effects of international trade and technical change on labour markets to avoid the limitations accompanied by using neoclassical models and the direct implementation of empirical analysis.

A Appendix

A.1 Cost Functions of a Two Sector and Two Factor Economy

Based on equations (5) the factor price ratio of sector 1 reveals as

$$\frac{w_1}{w_2} = \frac{\alpha_{11}}{\alpha_{12}} \frac{V_{12}}{V_{11}} \tag{33}$$

and based on equations (6) the factor price ratio of sector 2 reveals as

$$\frac{w_1}{w_2} = \frac{\alpha_{21}}{\alpha_{22}} \frac{V_{22}}{V_{21}}. \tag{34}$$

Hence, factor intensity of sector 1 is given by

$$\frac{V_{12}}{V_{11}} = \frac{w_1}{w_2} \frac{\alpha_{12}}{\alpha_{11}} \tag{35}$$

and of sector 2 by

$$\frac{V_{22}}{V_{21}} = \frac{w_1}{w_2} \frac{\alpha_{22}}{\alpha_{21}}. \tag{36}$$

Reinserting factor intensity (35) into (5) yields

$$\begin{aligned} w_1 &= p_1 A_1 \alpha_{11} \left(\frac{w_1}{w_2} \frac{\alpha_{12}}{\alpha_{11}} \right)^{\alpha_{12}} \\ w_2 &= p_1 A_1 \alpha_{12} \left(\frac{w_2}{w_1} \frac{\alpha_{11}}{\alpha_{12}} \right)^{\alpha_{11}}. \end{aligned} \tag{37}$$

Hence, by rearranging, the cost function of sector 1 reveals as

$$p_1 = \frac{1}{A_1} \frac{w_1^{(1-\alpha_{12})} w_2^{\alpha_{12}}}{\alpha_{12}^{\alpha_{12}} (1-\alpha_{12})^{(1-\alpha_{12})}}. \tag{38}$$

Reinserting factor intensity (36) into (6) yields

$$\begin{aligned} w_1 &= p_2 A_2 \alpha_{21} \left(\frac{w_1}{w_2} \frac{\alpha_{22}}{\alpha_{21}} \right)^{\alpha_{22}} \\ w_2 &= p_2 A_2 \alpha_{22} \left(\frac{w_2}{w_1} \frac{\alpha_{21}}{\alpha_{22}} \right)^{\alpha_{21}} \end{aligned} \tag{39}$$

Hence, by rearranging, the cost function of sector 2 reveals as

$$p_2 = \frac{1}{A_2} \frac{w_2^{(1-\alpha_{21})} w_1^{\alpha_{21}}}{\alpha_{21}^{\alpha_{21}} (1-\alpha_{21})^{(1-\alpha_{21})}}. \tag{40}$$

A.2 Determination of Factor Prices of a Two Sector and Two Factor Economy

The equation system (8) reveals in matrix notation as

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} \ln w_1 \\ \ln w_2 \end{pmatrix} = \begin{pmatrix} \ln \overline{p_1} \\ \ln \overline{p_2} \end{pmatrix}. \tag{41}$$

Solving by Cramer's Rule yields

$$\begin{aligned} \ln w_1 = \frac{Det(\underline{\Phi_1})}{Det(\underline{\Phi})} &= \frac{\ln \overline{p_1} \cdot \alpha_{22} - \ln \overline{p_2} \cdot \alpha_{12}}{\alpha_{11} \cdot \alpha_{22} - \alpha_{21} \cdot \alpha_{12}} = \frac{\ln \overline{p_1} \cdot \Phi_{11} + \ln \overline{p_2} \cdot \Phi_{21}}{\alpha_{11} \cdot \alpha_{22} - \alpha_{21} \cdot \alpha_{12}} \\ \ln w_2 = \frac{Det(\underline{\Phi_2})}{Det(\underline{\Phi})} &= \frac{\ln \overline{p_2} \cdot \alpha_{11} - \ln \overline{p_1} \cdot \alpha_{21}}{\alpha_{11} \cdot \alpha_{22} - \alpha_{21} \cdot \alpha_{12}} = \frac{\ln \overline{p_2} \cdot \Phi_{22} + \ln \overline{p_1} \cdot \Phi_{12}}{\alpha_{11} \cdot \alpha_{22} - \alpha_{21} \cdot \alpha_{12}} \end{aligned} \tag{42}$$

$$\begin{aligned} Det(\underline{\Phi}) \cdot \ln w_1 &= \sum_{i=1}^{2} \Phi_{i1} \cdot \ln \overline{p_i} \\ Det(\underline{\Phi}) \cdot \ln w_2 &= \sum_{i=1}^{2} \Phi_{i2} \cdot \ln \overline{p_i}, \end{aligned} \tag{43}$$

whereby determinants are given by

$$\begin{aligned} Det \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} &= Det(\underline{\Phi}) = \Phi \\ Det \begin{pmatrix} \ln \overline{p_1} & \alpha_{12} \\ \ln \overline{p_2} & \alpha_{22} \end{pmatrix} &= Det(\underline{\Phi_1}) \\ Det \begin{pmatrix} \alpha_{11} & \ln \overline{p_1} \\ \alpha_{21} & \ln \overline{p_2} \end{pmatrix} &= Det(\underline{\Phi_2}), \end{aligned} \tag{44}$$

with adjoints $\Phi_{11} = \alpha_{22}$, $\Phi_{21} = -\alpha_{12}$, $\Phi_{12} = -\alpha_{21}$ and $\Phi_{22} = \alpha_{11}$. Hence, the factor price of the economy reveals for factor 1 as

$$\begin{aligned} \ln w_1 &= \frac{1}{Det(\underline{\Phi})} \sum_{i=1}^{2} \Phi_{i1} \cdot \ln \overline{p_i} \\ &= \sum_{i=1}^{2} \frac{\Phi_{i1}}{\Phi} \ln A_i + \sum_{i=1}^{2} \frac{\Phi_{i1}}{\Phi} \ln p_i + \frac{1}{\Phi} \prod_{1=1}^{2} \left[\prod_{k=1}^{2} \alpha_{ik}^{\alpha_{ik}} \right]^{\Phi_{i1}} \\ &= \sum_{i=1}^{2} \beta_{i1} \ln A_i + \sum_{i=1}^{2} \beta_{i1} \ln p_i + \gamma_1 \end{aligned} \tag{45}$$

and for factor 2 as

$$\begin{aligned} \ln w_2 &= \frac{1}{Det(\underline{\Phi})} \sum_{i=1}^{2} \Phi_{i2} \cdot \ln \overline{p_i} \\ &= \sum_{i=1}^{2} \frac{\Phi_{i2}}{\Phi} \ln A_i + \sum_{i=1}^{2} \frac{\Phi_{i2}}{\Phi} \ln p_i + \frac{1}{\Phi} \prod_{1=1}^{2} \left[\prod_{k=1}^{2} \alpha_{ik}^{\alpha_{ik}} \right]^{\Phi_{i2}} \end{aligned}$$

$$= \sum_{i=1}^{2} \beta_{i2} \ln A_i + \sum_{i=1}^{2} \beta_{i2} \ln p_i + \gamma_2. \quad (46)$$

A.3 Cost Functions of a M Sector and N Factor Economy

Based on equations (15) the factor price ratios reveal as

$$\frac{w_k}{w_l} = \frac{\alpha_{ik}}{\alpha_{il}} \frac{V_{il}}{V_{ik}}. \quad (47)$$

Thus, factor intensities are given by

$$\frac{V_{il}}{V_{ik}} = \frac{w_k}{w_l} \frac{\alpha_{il}}{\alpha_{ik}}. \quad (48)$$

Reinserting the factor intensities (48) into equations (15) yields

$$w_k = p_i A_i \alpha_{ik} \Pi_{j=1}^{n} \left(\frac{w_k}{w_j} \frac{\alpha_{ij}}{\alpha_{ik}} \right)^{\alpha_{ij}}. \quad (49)$$

With $\Pi \alpha_{ik}^{\alpha_{ij}} = \alpha_{ik}^{\Sigma \alpha_{ij}} = \alpha_{ik}$ and $\Pi w_k^{\alpha_{ij}} = w_k^{\Sigma \alpha_{ij}} = w_k$ the cost functions reveal as

$$p_i = \frac{1}{A_i} \Pi_{j=1}^{n} \frac{w_j^{\alpha_{ij}}}{\alpha_{ij}^{\alpha_{ij}}}. \quad (50)$$

A.4 Determination of Factor Prices of a M Sector and N Factor Economy

With $\ln w_j = \nu_j$ and $\ln \overline{p_j} = \eta_j$ the system (17) in matrix notation reveals as

$$\underline{\Phi} \underline{v} = \underline{\eta}. \quad (51)$$

$\underline{\Phi}$ is the matrix with $\alpha-$ components, ν_j the unknowns and η_j the right hand side. The matrix $\underline{\Phi}$ is nonsingular. Solving (51) by Cramer's Rule yields

$$\nu_i = \frac{1}{Det(\underline{\Phi})} Det(\underline{\Phi_i}). \quad (52)$$

Using $\underline{\Phi}$ and substituting the ith-column by the right hand side of (51), denoting the result $\underline{\Phi_i}$, it follows for case $i = 1$

$$\ln w_1 = \frac{1}{\det(\underline{\Phi})} \det \begin{pmatrix} \ln \overline{p_1} & \alpha_{12} & .. & \alpha_{1n} \\ \ln \overline{p_2} & \alpha_{22} & .. & \alpha_{2n} \\ .. & .. & .. & .. \\ \ln \overline{p_n} & \alpha_{n2} & .. & \alpha_{nn} \end{pmatrix}. \quad (53)$$

Expansion of the first column yields

$$\det(\underline{\Phi}) \ln w_1 = \ln \overline{p_1} \cdot \Phi_{11} + \ln \overline{p_2} \cdot \Phi_{21} + ... + \ln \overline{p_n} \cdot \Phi_{n1} = \sum_{i=1}^{n} \Phi_{i1} \ln \overline{p_i}$$

$$\ln w_1^{\det(\underline{\Phi})} = \sum_{i=1}^{n} \ln \overline{p_i}^{\Phi_{i1}}. \tag{54}$$

Hence, the general solution of the equation system (17) reveals as

$$\det(\underline{\Phi}) \ln w_j = \ln \overline{p_1} \cdot \Phi_{1j} + \ln \overline{p_2} \cdot \Phi_{2j} + ... + \ln \overline{p_n} \cdot \Phi_{nj} = \sum_{i=1}^{n} \Phi_{ij} \ln \overline{p_i}. \tag{55}$$

Rearranging (55) and denoting $\overline{p_i} = A_i p_i \Pi_{k=1}^{n} \alpha_{ik}^{\alpha_{ik}}$, $\Phi = \det(\underline{\Phi})$, $\beta_{ij} = \Phi_{ij}/\Phi$, $\alpha_j = \Pi_{i=1}^{n}(\Pi_{k=1}^{n} \alpha_{ik}^{\alpha_{ik}})^{\Phi_{ij}}$ and $\gamma_j = \frac{1}{\Phi} \ln \alpha_j$ the factor prices of the economy reveal as

$$\ln w_j = \gamma_j + \Sigma_{i=1}^{n} \beta_{ij} \ln A_i + \Sigma_{i=1}^{n} \beta_{ij} \ln p_i. \tag{56}$$

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